

NAG Toolbox for MATLAB

f07hg

1 Purpose

f07hg estimates the condition number of a real symmetric positive-definite band matrix A , where A has been factorized by f07hd.

2 Syntax

```
[rcond, info] = f07hg(uplo, kd, ab, anorm, 'n', n)
```

3 Description

f07hg estimates the condition number (in the 1-norm) of a real symmetric positive-definite band matrix A :

$$\kappa_1(A) = \|A\|_1 \|A^{-1}\|_1.$$

Since A is symmetric, $\kappa_1(A) = \kappa_\infty(A) = \|A\|_\infty \|A^{-1}\|_\infty$.

Because $\kappa_1(A)$ is infinite if A is singular, the function actually returns an estimate of the **reciprocal** of $\kappa_1(A)$.

4 References

Higham N J 1988 FORTRAN codes for estimating the one-norm of a real or complex matrix, with applications to condition estimation *ACM Trans. Math. Software* **14** 381–396

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo** – **string**

Indicates how A has been factorized.

uplo = 'U'

$A = U^T U$, where U is upper triangular.

uplo = 'L'

$A = L L^T$, where L is lower triangular.

Constraint: **uplo** = 'U' or 'L'.

2: **kd** – **int32 scalar**

k_d , the number of superdiagonals or subdiagonals of the matrix A .

Constraint: **kd** ≥ 0 .

3: **ab(ldab,*)** – **double array**

The first dimension of the array **ab** must be at least **kd** + 1

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The Cholesky factor of A , as returned by f07hd.

4: **anorm** – double scalar

The 1-norm of the **original** matrix A , which may be computed by calling Missing 'id'. **anorm** must be computed either **before** calling f07hd or else from a copy of the original matrix A .

Constraint: **anorm** ≥ 0.0 .

5.2 Optional Input Parameters1: **n** – int32 scalar

Default: The second dimension of the array **ab**.

n , the order of the matrix A .

Constraint: **n** ≥ 0 .

5.3 Input Parameters Omitted from the MATLAB Interface

ldab, work, iwork

5.4 Output Parameters1: **rcond** – double scalar

An estimate of the reciprocal of the condition number of A . **rcond** is set to zero if exact singularity is detected or the estimate underflows. If **rcond** is less than *machine precision*, A is singular to working precision.

2: **info** – int32 scalar

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info = $-i$

If **info** = $-i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **kd**, 4: **ab**, 5: **ldab**, 6: **anorm**, 7: **rcond**, 8: **work**, 9: **iwork**, 10: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

The computed estimate **rcond** is never less than the true value ρ , and in practice is nearly always less than 10ρ , although examples can be constructed where **rcond** is much larger.

8 Further Comments

A call to f07hg involves solving a number of systems of linear equations of the form $Ax = b$; the number is usually 4 or 5 and never more than 11. Each solution involves approximately $4nk$ floating-point operations (assuming $n \gg k$) but takes considerably longer than a call to f07he with one right-hand side, because extra care is taken to avoid overflow when A is approximately singular.

The complex analogue of this function is f07hu.

9 Example

```
uplo = 'L';  
kd = int32(1);  
ab = [2.343074902771996, 2.078877201506509, 1.130612248337004,  
1.146524711734229;  
1.143796127400537, -1.149659055507477, -1.963537900164584,  
5.495816452771857e+222];  
anorm = 10.7;  
[rcond, info] = f07hg(uplo, kd, ab, anorm)  
  
rcond =  
0.0135  
info =  
0
```
